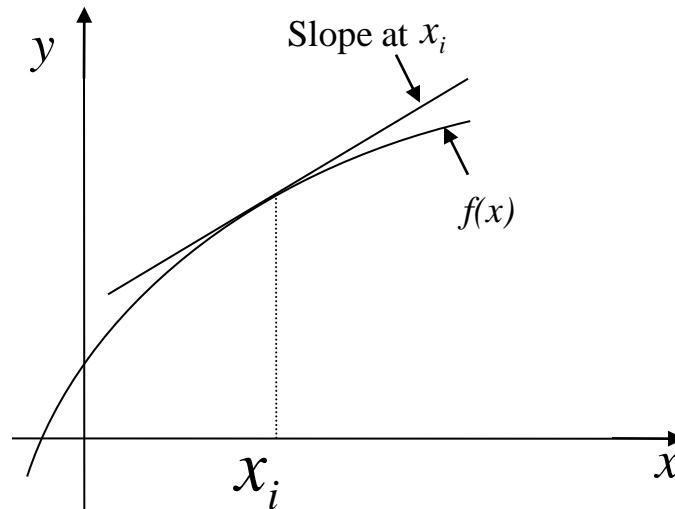


# NUMERICAL METHODS

## Forward Divided Difference

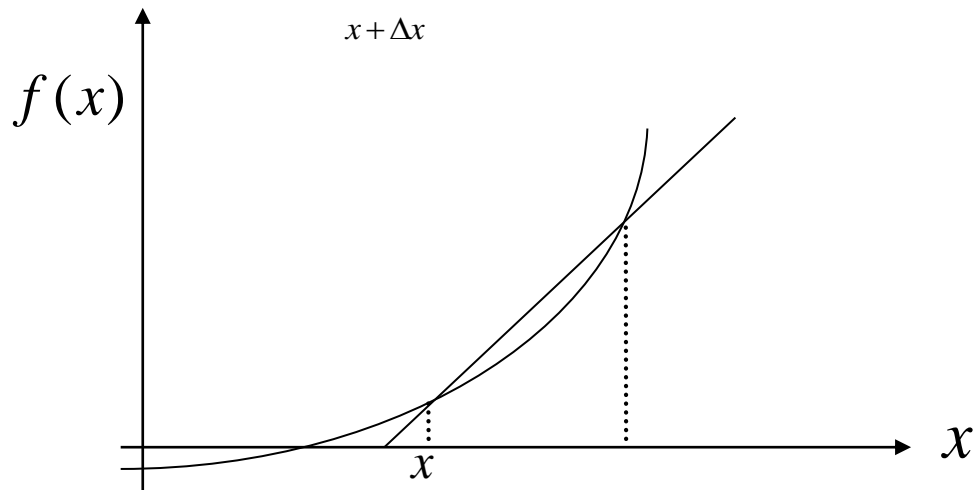
# Definition

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



# Forward Divided Difference

$$f'(x) \cong \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



$$f'(x_i) \cong \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

# Example

Example:

The velocity of a rocket is given by

$$v(t) = 2000 \ln \left[ \frac{14 \times 10^4}{14 \times 10^4 - 2100t} \right] - 9.8t, 0 \leq t \leq 30$$

where  $v$  given in m/s and  $t$  is given in seconds. Use forward difference approximation of the first derivative of  $v(t)$  to calculate the acceleration at  $t = 16s$ . Use a step size of  $\Delta t = 2s$ .

Solution:

$$a(t_i) \cong \frac{v(t_{i+1}) - v(t_i)}{\Delta t}$$

$$t_i = 16$$

$$\Delta t = 2$$

$$t_{i+1} = t_i + \Delta t = 16 + 2 = 18$$

$$a(16) = \frac{v(18) - v(16)}{2}$$

$$v(18) = 2000 \ln \left[ \frac{14 \times 10^4}{14 \times 10^4 - 2100(18)} \right] - 9.8(18) = 453.02 \text{ m/s}$$

$$v(16) = 2000 \ln \left[ \frac{14 \times 10^4}{14 \times 10^4 - 2100(16)} \right] - 9.8(16) = 392.07 \text{ m/s}$$

Hence

$$a(16) = \frac{v(18) - v(16)}{2} = 453.02 - 392.07 = 30.475 \text{ m/s}^2$$

The exact value of  $a(16)$  can be calculated by differentiating

$$v(t) = 2000 \ln \left[ \frac{14 \times 10^4}{14 \times 10^4 - 2100t} \right] - 9.8t$$

as

$$a(t) = \frac{d}{dt} [v(t)] = \frac{-4040 - 29.4t}{-200 + 3t}$$

$$a(16) = 29.674 \text{ m/s}^2$$

The absolute relative true error is

$$\begin{aligned} |\varepsilon_t| &= \left| \frac{\text{True Value} - \text{Approximate Value}}{\text{True Value}} \right| \times 100 \\ &= \left| \frac{29.674 - 30.475}{29.674} \right| \times 100 \\ &= 2.6993\% \end{aligned}$$

# Effect Of Step Size

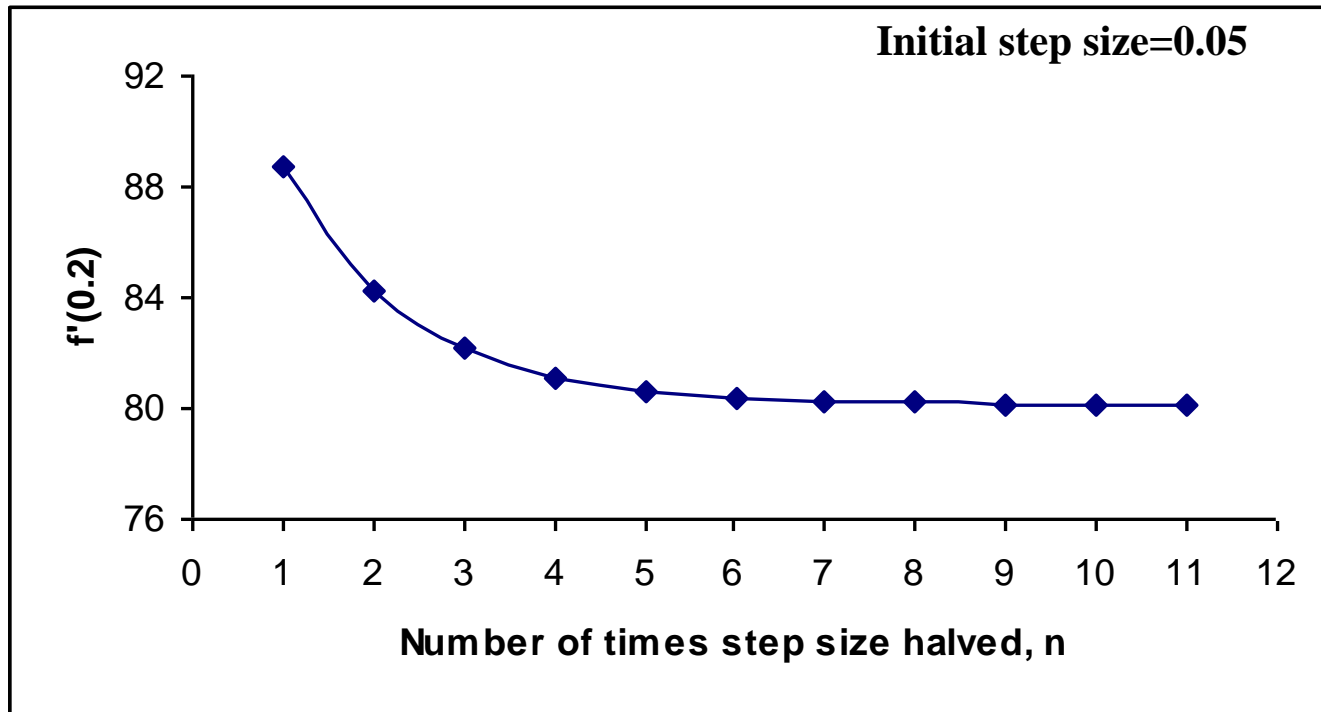
$$f(x) = 9e^{4x}$$

Value of  $f'(0.2)$  Using forward difference method.

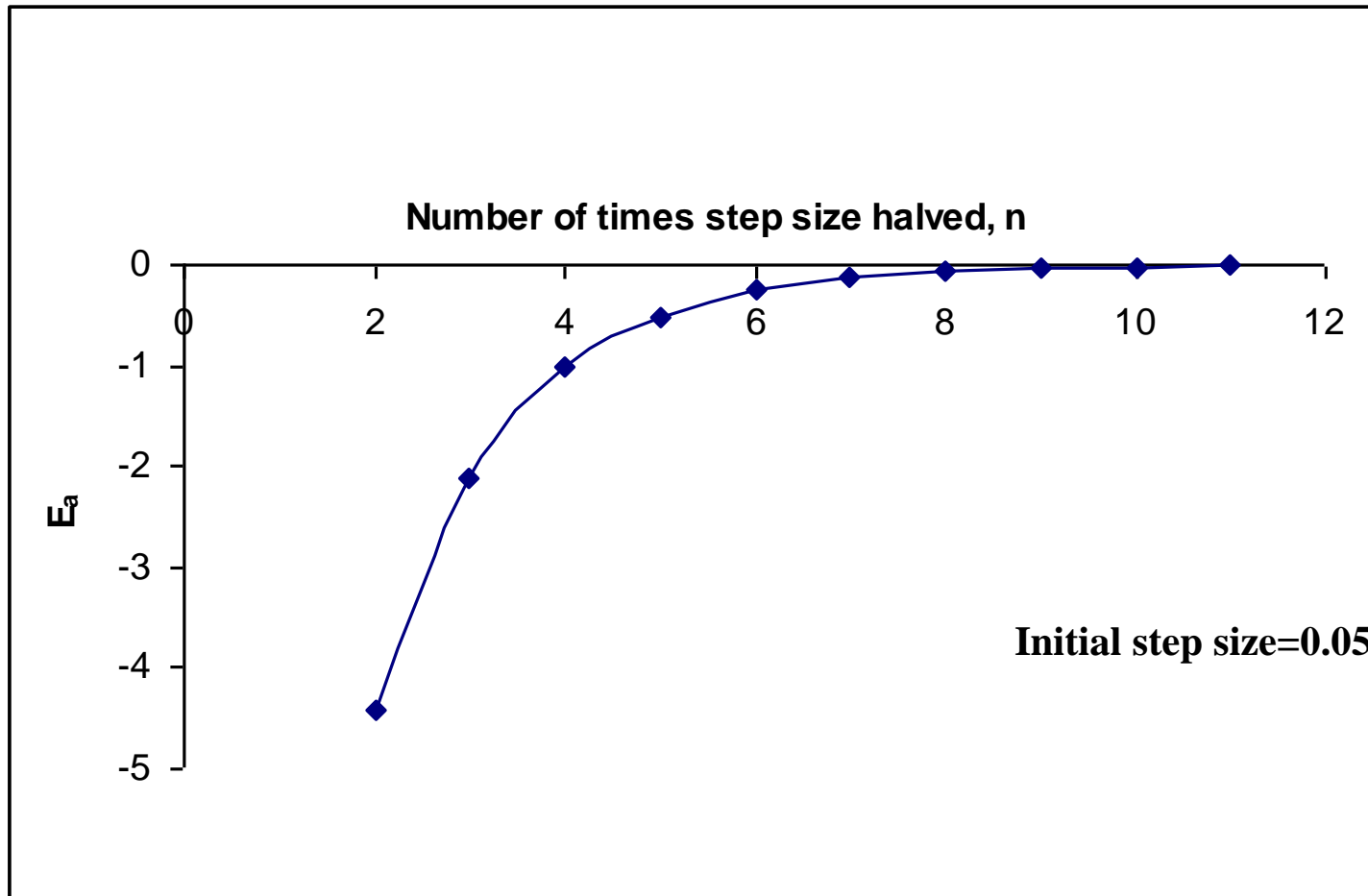
h	$f'(0.2)$	$E_a$	$ \varepsilon_a  \%$	Significant digits	$E_t$	$ \varepsilon_t  \%$
0.05	88.69336				-8.57389	10.70138
0.025	84.26239	-4.430976	5.258546	0	-4.14291	5.170918
0.0125	82.15626	-2.106121	2.563555	1	-2.03679	2.542193
0.00625	81.12937	-1.0269	1.265756	1	-1.00989	1.260482
0.003125	80.62231	-0.507052	0.628923	1	-0.50284	0.627612
0.001563	80.37037	-0.251944	0.313479	2	-0.25090	0.313152
0.000781	80.24479	-0.125579	0.156494	2	-0.12532	0.156413
0.000391	80.18210	-0.062691	0.078186	2	-0.06263	0.078166
0.000195	80.15078	-0.031321	0.039078	3	-0.03130	0.039073
9.77E-05	80.13512	-0.015654	0.019535	3	-0.01565	0.019534
4.88E-05	80.12730	-0.007826	0.009767	3	-0.00782	0.009766



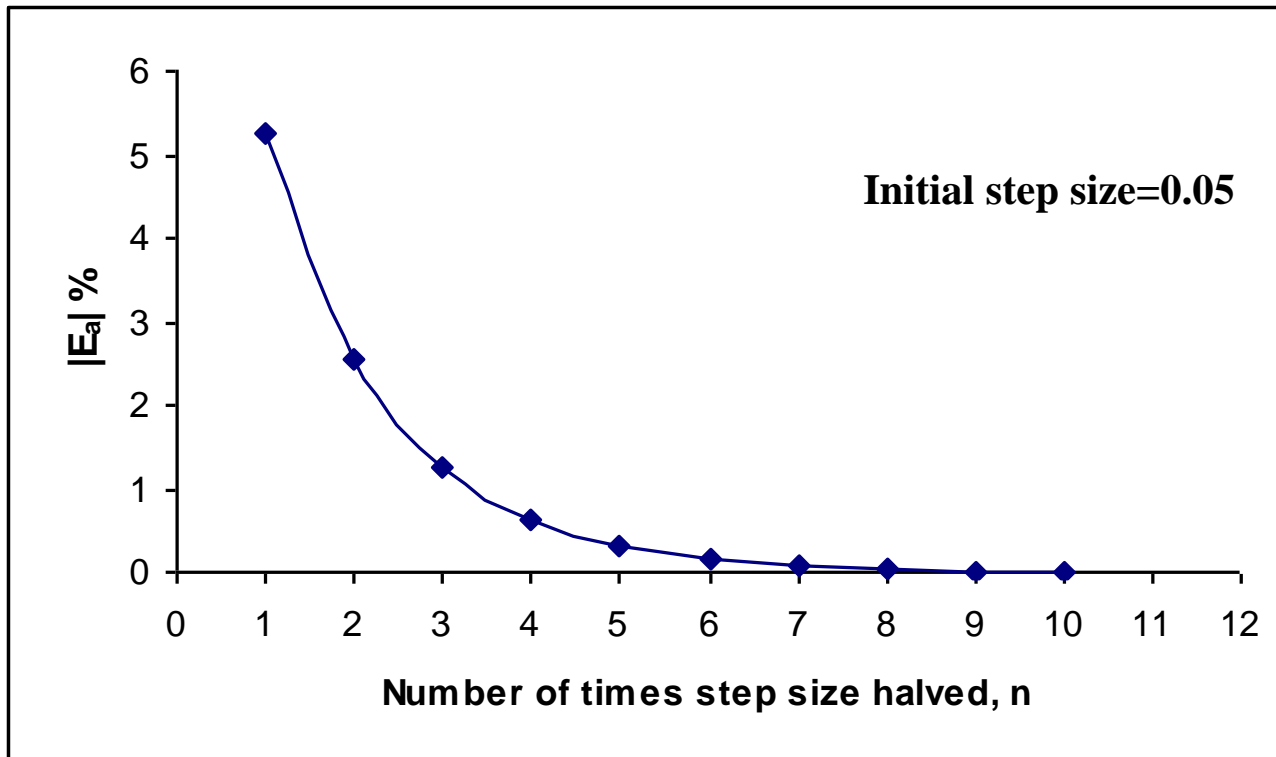
# Effect of Step Size in Forward Divided Difference Method



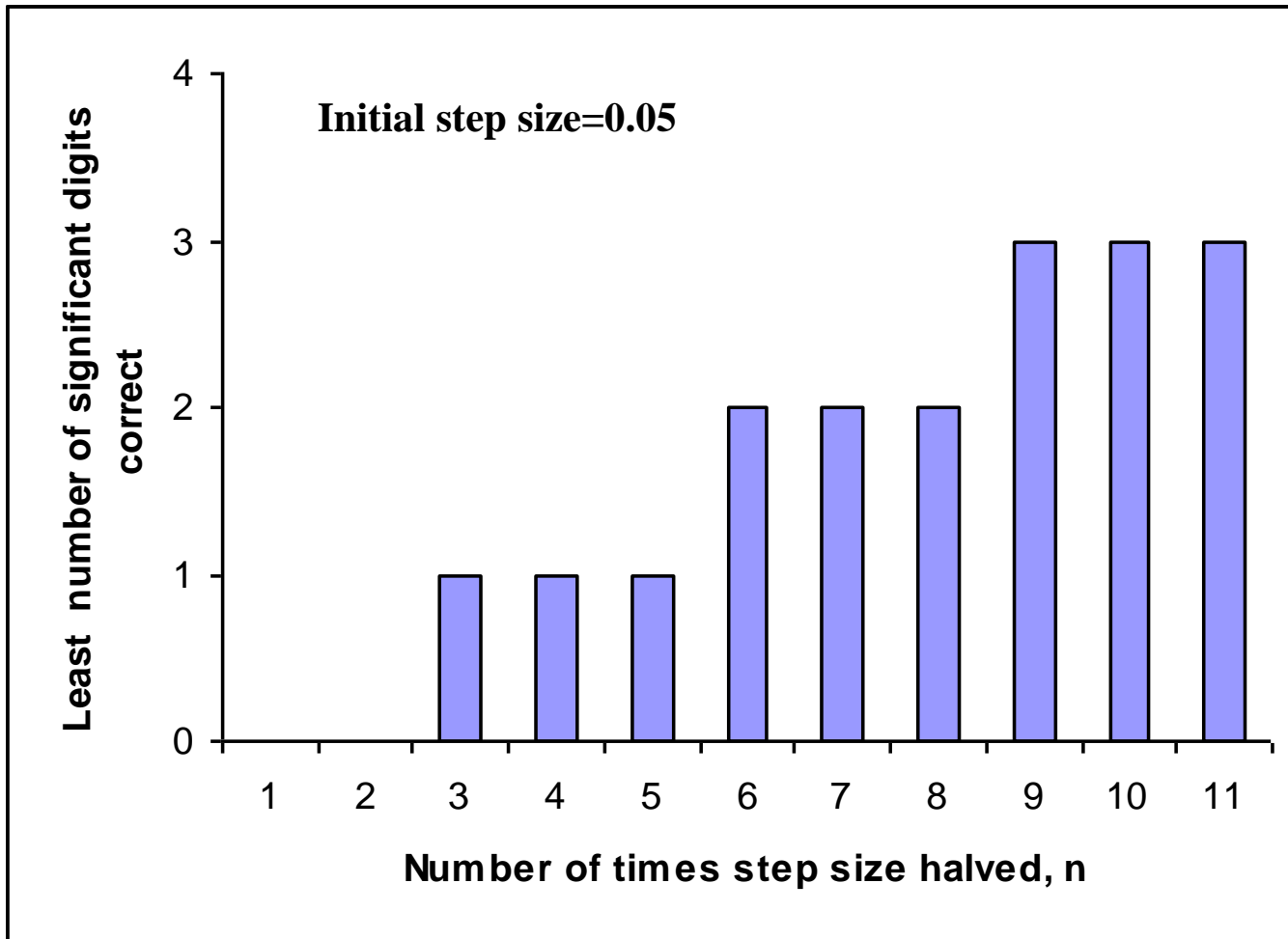
# Effect of Step Size on Approximate Error



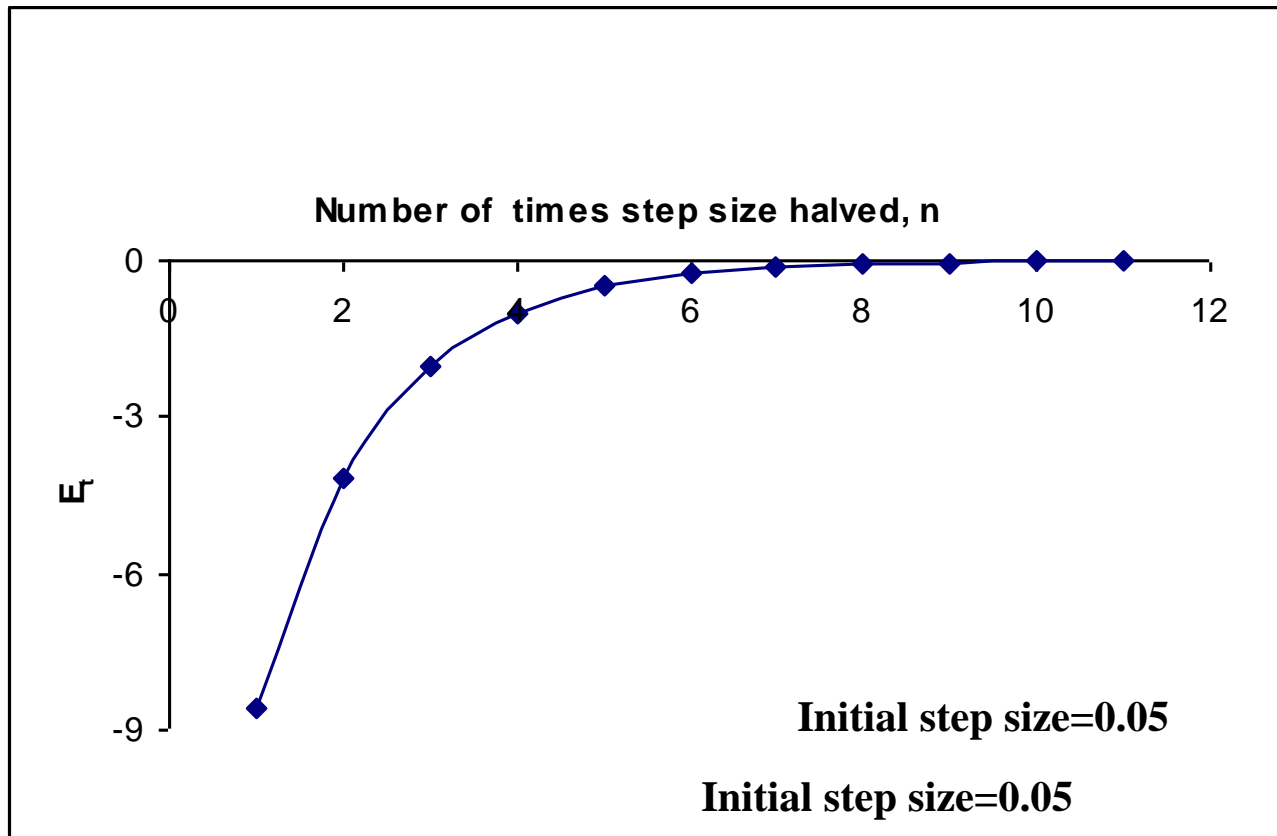
# Effect of Step Size on Absolute Relative Approximate Error



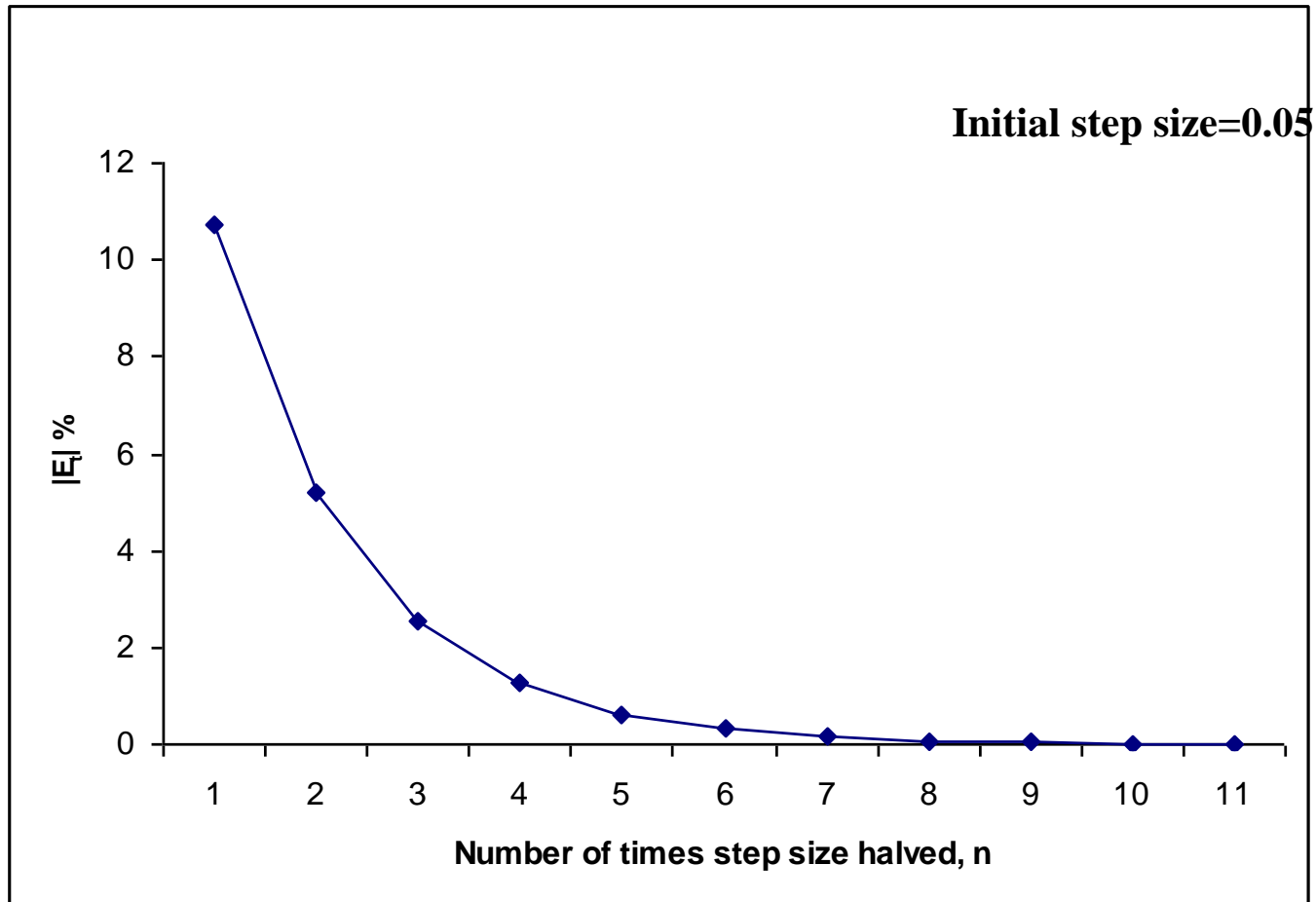
# Effect of Step Size on Least Number of Significant Digits Correct



# Effect of Step Size on True Error



# Effect of Step Size on Absolute Relative True Error



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